J. Fluid Mech. (1971), *vol.* 49, *part* 2, *pp.* 399–414
Printed in Great Britain

On combustion generated noise

By WARREN C. STRAHLE
Georgia Institute of Technology

(Received 7 January 1971)

Upon review of past experimental results and theoretical efforts it is apparent that the mechanism by which combustion noise is generated is not well understood. A theory of combustion noise is developed in this paper which follows rigorously from the principles of fluid mechanics. Lighthill's approach, used in his studies of aerodynamic noise, is closely followed in the present work. The sound radiated from open, turbulent flames is found to depend strongly upon the structure of such flames; at present their structure is not well known. However, meaningful bounds and scaling rules for the sound power output and spectral content are derived based upon the present limited knowledge. A framework is developed which explains past experimental work and the origin of combustion noise.

1. Introduction

Combustion noise considered as an identifiable phenomenon, separate from other noise sources, has received only scattered attention in the literature. An excellent review of the experimental work in this area has been given by Giammar & Putnam (1970). The work on combustion noise has been primarily oriented toward open premixed or open diffusion flames or industrial burners. Turbulent flames have been emphasized. There has been virtually no research effort in the area of combustion noise generated in turbo-propulsion systems for aircraft.

Combustion noise may be categorized as either direct or indirect. An example of indirect noise is the increase in jet noise caused by a higher jet velocity after passage of a turbojet flow through an afterburner. As another example, the noise radiated from a diesel engine is indirectly caused by the combustion process because of the periodic pressure rise caused by combustion in each cylinder. Of interest in this paper is direct combustion noise which has its origin in and is radiated from a combustion region.

From the summary given by Giammar & Putnam (1970) several conclusions may be drawn concerning combustion noise, not the least of which is the fact that combustion noise exists as an identifiable noise source, independent of jet noise. Combustion noise fills the lower portion of the audible frequency spectrum ($\lesssim$ 1500 Hz) and has a characteristic frequency of maximum sound output which depends upon both the flow velocity and reaction chemistry, as shown by Smith & Kilham (1963). In its appropriate frequency range combustion noise overwhelms jet noise. The sound power radiated from turbulent flames has distinctly different, but less precisely known or understood, scaling rules as compared with jet noise. There are no clear differences in behaviour between premixed and diffusion flames. The sound power output varies as the flow velocity to an exponent between one and four, with the characteristic flow dimension to a power between one and three and with the laminar flame speed, representative of the chemistry effect, to an exponent between $\frac{1}{2}$ and $3\frac{1}{2}$. An important characteristic is the apparent monopole source behaviour; that is, there is little directionality to the radiated sound field. All of the above comments relate to subsonic combustion and subsonic jets. There exist no data on supersonic combustion noise.

Theoretical work concerning combustion noise is in a state of infancy, with the existence of only one plausible method for the estimation of sound power output from a combustion region. The theory developed by Bragg (1963) is a rather ingenious development on the basis of physical reasoning using the wrinkled laminar flame concept as developed by Karlovitz (1951) and others. According to the wrinkled flame model of the turbulent combustion process the flame propagates by a locally laminar mechanism. The only effect of turbulence is to distort the flame surface area. Bragg then compares the consumption of the turbulent eddies to the expansion of fluid elements acting as monopole acoustic sources. The major result of the theory is an expression for the thermo-acoustic efficiency, η_{ta}, which measures the ratio of sound power output to the combustion energy release rate. The theory has the virtues that the magnitude of the sound output is roughly in accord with experiment, within a factor of 100 for published noise data, the output is omnidirectional by assumption, and the scaling with flow and chemistry variables is within the ranges discussed above. However, the theory has at least two major deficiencies. First, the theory does not follow rigorously from the principles of fluid mechanics and it rests upon a model of the turbulent flame which is open to question (John & Summerfield 1957). It will be shown below that the theory is not compatible with the equations of fluid mechanics even assuming the wrinkled flame model is valid. Second, the rather wide observed variation in scaling rules mentioned above is not explained by the theory. Consequently, the noise output can be estimated incorrectly by several orders of magnitude, depending upon the flow variables, size scale, and chemistry. The present theory attempts to clarify the rigorous basis of Bragg's theory and to derive a more accurate description of combustion noise.

Another theoretical effort has been reported by Kotake & Hatta (1965). In that work an inhomogeneous wave equation was derived which differed from the Lighthill (1952) formulation of the noise problem. The major differences are accounted for by the introduction of the first and second laws of thermodynamics as well as the only two relations of the Lighthill formulation, continuity and momentum conservation. This procedure leads to a severe complication of the equations and prevents a simple interpretation of combustion noise. In addition an erroneous conclusion is drawn that the sound power should be proportional to the flow velocity, U, to the fourth power, and this is interpreted as dipole radiation produced by the jet. As clearly shown by Lighthill, this cannot be the case for subsonic jets. The U^4 scaling, which has been observed in some combustion noise experiments, including those of Kotake & Hatta, can be deduced as appropriate for combustion noise using the developments of the present paper. However, such noise is not properly ascribable to the jet. In any event Kotake & Hatta did not arrive at a formula for estimation of the sound power output so the theory is not useful in the sense of Bragg's theory.

Because of the apparent lack of understanding of combustion noise, the purpose of the present paper is to construct a theory capable of explanation of the experimental facts. It will be found possible to do so; however, the lack of knowledge of turbulent flame structure will cause much less precision in the results as compared with jet noise theory.

2. Analysis of the acoustics

Consider the configuration of figure 1 in which there is a well-defined acoustic source region containing violent turbulent motion. The sound generated in this region is radiated to the surroundings which are of infinite extent and quiescent.

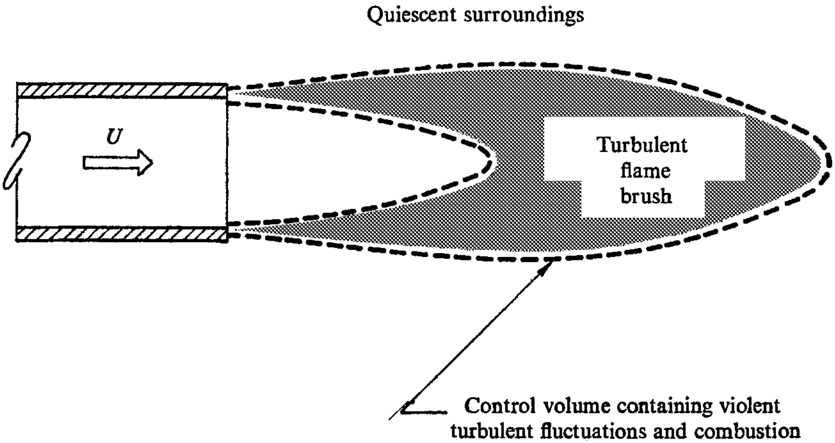

FIGURE 1. Configuration schematic.

The region of turbulent motion contains a flame which may either be a premixed flame or a diffusion flame. Following Lighthill (1952), the continuity and momentum equations may be combined to form an inhomogeneous wave equation of the form

$$\rho_{tt} - a_0^2 \rho_{x_j x_j} = (f_{ij})_{x_i x_j} \equiv F, \tag{1}$$

where ρ is the fluid density, a_0 is the (uniform) speed of sound of the quiescent surroundings; t is time, x_i are the space co-ordinates, subscript by an independent variable denotes a partial derivative and f_{ij} is defined by

$$f_{ij} = \rho v_i v_j + (p - a_0^2 \rho)\, \delta_{ij}. \tag{2}$$

In (2) v_i is the i component of the fluid velocity, p is the pressure and δ_{ij} is the Kronecker delta. The viscous contribution to the stress tensor has been omitted from (2) following Lighthill's (1952) arguments, which are also valid in the presence of a flame. The right-hand side of (1) may be split into three components using (2) to obtain

$$\left.\begin{aligned} F_1 &= (\rho v_i v_j)_{x_i x_j}, \quad F_2 = p_{x_j x_j}, \\ F_3 &= -a_0^2 \rho_{x_j x_j}, \quad F = F_1 + F_2 + F_3. \end{aligned}\right\} \tag{3}$$

Relations (1) and (3) may be viewed as valid for the fluctuations of ρ, F_1, F_2 and F_3 about their mean values by the procedure of splitting these quantities into mean plus fluctuating components and performing a long time average which is then subtracted from (1). Consequently, in the following relations (1) and (3) will be considered in terms of fluctuating quantities. It is to be emphasized that (1) is exact; no linearization has been performed.

If the right-hand side of (1) is known, it may be solved exactly for the density, if the effect of the pipe on the radiated sound is neglected. Of course, as has been discussed at length by Lighthill (1952), Ribner (1964) and Curle (1968) the right-hand side contains the unknown. Furthermore, because of the presence of p and v_i, (1) contains more unknowns than ρ itself. However, if the right-hand side of (1) were known from experiment, say, it follows that the solution to (1) must be in accord with reality since both the physical laws constituting (1) and the experimental results describing the inhomogeneity are expressions of physical reality.

In order to gain a solution to (1) a controversial procedure will be applied. It is the same procedure used by Lighthill (1952), but the method certainly contains some troublesome operations. The argument used is that first of all the turbulent fluctuations within the turbulent region are substantially more violent than the acoustic fluctuations which are produced. A straightforward perturbation scheme suggests the first approximation to the solution is to consider the fluctuations in F to be those due to turbulent motion and not sound propagation within the turbulent zone; furthermore, F is considered zero outside the source region. Thus, as a first approximation, the effects of the sound field upon F itself would not need to be considered if the above reasoning were correct. The solution to (1), which is an inhomogeneous wave equation, would then be given by

$$\rho(\mathbf{x},t) = \frac{1}{4\pi a_0^2}\int_V F\left(\mathbf{y}, t-\frac{|\mathbf{x}-\mathbf{y}|}{a_0}\right)\frac{d\tau(\mathbf{y})}{|\mathbf{x}-\mathbf{y}|}. \tag{4}$$

In (4) $\mathbf{x}$ is the position of observation. Presuming enough experimental data has been accumulated to determine F, (4), which is the solution in Lighthill's (1952) form, describes the sound field.

The above reasoning is correct if F does not contain any terms essential to the dynamics of sound propagation within the turbulent region. That is, terms describing complex refraction and reflection effects properly belong on the left-hand side of (1). This difficulty has been discussed at length by many workers, most recently by Doak (1970), and it appears that, indeed, (4) is inexact. But it still appears an open issue whether or not (4) is sufficient for an order-of-magnitude analysis of the radiated sound and for a deduction of some of the important characteristics of the sound field, such as directional radiation properties. In view of the success of the Lighthill (1952) theory for these estimation purposes for jet noise, it appears that no major error is committed in (4).

For the case of reacting gases, which are being treated in this paper, additional difficulties arise. The well-known phenomenon of combustion instability, as put forth by Crocco & Cheng (1956), for example, implies strong coupling between a sound field and the combustion processes. As a simple example it is well known from chemical kinetics that reaction rates are pressure and temperature sensitive.

It is also known that flames are sensitive to acoustic disturbances as shown by Briffa & Fursey (1967). These facts imply that F cannot be solely determined from the turbulence properties, independently of the sound field. Nevertheless, as mentioned above, it is possible that (4) gives a reasonable description on an order of magnitude basis for the radiated sound, assuming F is determined by turbulence properties alone. The philosophy adopted in this paper is to presume (4) is valid, carry through the analysis to determine the characteristics of the sound field and to then compare the results with available experiments. It will be found below that no serious error is apparently committed in using (4).

The solution may be split into the sum of three volume integrals corresponding to the three terms of (3). In Lighthill's (1952) theory for jets with a sound speed equal to that of the undisturbed fluid the integrals containing F_2 and F_3 cancelled for adiabatic changes in the field quantities p and ρ. For either hot or cold jets Lighthill (1954) showed that the contributions due to F_2 and F_3 were negligible compared to that due to F_1, again presuming an adiabatic relation between the p and ρ fluctuations. Consequently, the now classical quadrupole source due to F_1 was the only sound contributor in the original developments. Furthermore, since changes in ρ are small for low Mach number flows and small fluctuations, all changes in F_1 are due to $v_i v_j$, a quantity well investigated experimentally for subsonic jets. From (4), therefore, an important theory of jet noise based upon F_1 emerged, and it was the first explanation of the U^8 law for the radiated sound power as well as for other characteristics of the radiated sound field.

In the present work it may be taken as an experimental fact that the sound due to F_1 is dominated by some other source, at least in the lower frequency régime, because F_1 leads to a U^8 sound power output scaling. This scaling is not observed in combustion noise. Furthermore, the directionality of sound generated aerodynamically in the presence of a large mean shear is not observed in combustion noise. F_1 will consequently be neglected in the following. In a flame zone p is not related adiabatically to ρ and consequently Lighthill's treatment of F_2 and F_3 is inapplicable here. Consider the term due to F_3 first by splitting ρ into two parts, $\rho^{(2)}+\rho^{(3)}$, where by definition

$$\left.\begin{aligned} \rho^{(3)} &= \frac{1}{4\pi a_0^2}\int_V F_3(\mathbf{y},\eta)\,\frac{d\tau(\mathbf{y})}{|\mathbf{x}-\mathbf{y}|}, \\ \eta &= t-\frac{|\mathbf{x}-\mathbf{y}|}{a_0}. \end{aligned}\right\} \tag{5}$$

It is now desirable to perform a few operations on (5) which will better elucidate the origin of combustion noise. In almost a parenthetical comment Lighthill (1952) performed similar manipulations which have since given many students of the subject considerable difficulty. Curle (1968) attempted a detailed explanation of the manœuvres but there is unfortunately an error in his exposition. Consequently, at the risk of pedantry, considerable detail will be given in the following operations. Considering the makeup of $F_3 = -a_0^2\rho_{x_i x_i}$, it is desired to apply the divergence theorem to (5). The following relations will be useful:

$$\frac{\partial \eta}{\partial x_i} = -\frac{1}{a_0}\frac{\partial|\mathbf{x}-\mathbf{y}|}{\partial x_i} = \frac{1}{a_0}\frac{\partial|\mathbf{x}-\mathbf{y}|}{\partial y_i},$$

$$\left.\frac{\partial^2\rho(y_i,t')}{\partial y_i\partial y_i}\right|_{t'=\eta} = \left.\frac{\partial^2\rho(y_i,\eta)}{\partial y_i\partial y_i}\right|_{\eta=\text{constant}},$$

$$\frac{\partial\rho(y_i,\eta)}{\partial x_i} = -\frac{\partial\rho}{\partial\eta}\frac{1}{a_0}\frac{\partial|\mathbf{x}-\mathbf{y}|}{\partial x_i}.$$

It is now noted that

$$Z \equiv \int_V \rho_{y_iy_i}(y_i,\eta)\frac{d\tau}{|\mathbf{x}-\mathbf{y}|}$$

does not contain complete differentiation of the function ρ with respect to y_i because η contains a y_i dependence and η is explicitly held fixed in the above differentiation. Let $\zeta_i = \partial\rho/\partial y_i|_{\eta=\text{constant}}$. Then

$$\partial\zeta_i/\partial y_i|_{t=\text{constant}} = \partial\zeta_i/\partial y_i|_{\eta=\text{constant}} - \frac{1}{a_0}\frac{\partial|\mathbf{x}-\mathbf{y}|}{\partial y_i}\left.\frac{\partial\zeta_i}{\partial\eta}\right|_{y_i=\text{constant}}.$$

Therefore

$$Z = \int_V \frac{\partial}{\partial y_i}\left[\frac{\zeta_i}{|\mathbf{x}-\mathbf{y}|}\right]d\tau - \int_V \zeta_i\frac{\partial}{\partial y_i}\left(\frac{1}{|\mathbf{x}-\mathbf{y}|}\right)d\tau + \frac{1}{a_0}\int_V \frac{\partial\zeta_i}{\partial\eta}\frac{\partial|\mathbf{x}-\mathbf{y}|}{\partial y_i}\frac{d\tau}{|\mathbf{x}-\mathbf{y}|}.$$

The first term of Z is now clearly a divergence and may be replaced by a surface integral over the boundary of the region. But it has been presumed that outside of the violent region F, and consequently ζ_i, vanishes. Therefore, using the above identities

$$\begin{aligned} Z &= +\int_V \zeta_i\frac{\partial}{\partial x_i}\left(\frac{1}{|\mathbf{x}-\mathbf{y}|}\right) + \frac{1}{a_0}\int_V \frac{\partial\zeta_i}{\partial\eta}\frac{\partial|\mathbf{x}-\mathbf{y}|}{\partial y_i}\frac{d\tau}{|\mathbf{x}-\mathbf{y}|} \\ &= \int_V \frac{\partial}{\partial x_i}\left[\frac{\zeta_i}{|\mathbf{x}-\mathbf{y}|}\right]d\tau - \int_V \frac{\partial}{\partial x_i}[\zeta_i]\frac{d\tau}{|\mathbf{x}-\mathbf{y}|} - \int_V \frac{1}{a_0}\frac{\partial|\mathbf{x}-\mathbf{y}|}{\partial x_i}\frac{\partial\zeta_i/\partial\eta}{|\mathbf{x}-\mathbf{y}|}d\tau \\ &= \int_V \frac{\partial}{\partial x_i}\left[\frac{\zeta_i}{|\mathbf{x}-\mathbf{y}|}\right]d\tau = \frac{\partial}{\partial x_i}\int_V \frac{\zeta_i}{|\mathbf{x}-\mathbf{y}|}d\tau. \end{aligned}$$

Repeating this process yields

$$\rho^{(3)} = -\frac{1}{4\pi a_0^2}\frac{\partial^2}{\partial x_i^2}\int_V \frac{\rho(\mathbf{y},\eta)}{|\mathbf{x}-\mathbf{y}|}d\tau(\mathbf{y}).$$

Now carrying out the indicated differentiations and observing from the far field where $|\mathbf{x}-\mathbf{y}| = r \approx |\mathbf{x}|$, differentiation of $1/r$ with respect to x_i produces terms like $1/r^2$ and $1/r^3$ which are much smaller than the leading term

$$\rho^{(3)} = -\frac{1}{4\pi a_0^2}\frac{1}{r}\int_V \frac{\partial^2\rho}{\partial\eta^2}d\tau = -\frac{1}{4\pi a_0^2 r}\frac{\partial^2}{\partial t^2}\int_V \rho(\mathbf{y},\eta)\,d\tau. \tag{6}$$

In the far field η becomes nearly independent of $\mathbf{y}$. In fact

$$\eta = t - \frac{r}{a_0}\left[1 - \frac{x_iy_i}{r^2} + O\left(\frac{l^2}{r^2}\right)\right]$$

since x_i is $O(r)$ and y_i is $O(l)$ where l is a typical flow dimension. Expanding the integrand of (6)

$$\rho^{(3)} = -\frac{1}{4\pi a_0^2 r}\frac{\partial^2}{\partial t^2}\int_V \left\{\rho\left(\mathbf{y}, t-\frac{r}{a_0}\right) + \left.\frac{\partial\rho}{\partial\eta}\right|_{\eta=t-r/a_0}\frac{x_iy_i}{a_0 r} + \dots\right\}d\tau(\mathbf{y}). \tag{7}$$

The second term of the integrand is now $O(\omega l/a_0)$ compared to the first term since $\partial/\partial\eta$ is a typical frequency, ω, x_i is $O(r)$ and y_i is $O(l)$. Consequently, if the wave-length of the sound is sufficiently large compared to the macroscopic dimensions of the combustion field the effect of the variations in the retarded time may be neglected. This will be presumed to be the case, and in fact it corresponds to the situation for combustion noise experiments in the literature. It should be cautioned, however, that for sufficiently large burners the assumptions would be violated. Under the present assumption (7) becomes

$$\rho^{(3)} = -\frac{1}{4\pi a_0^2 r}\frac{\partial^2}{\partial t^2}\int_V \rho\left(\mathbf{y}, t-\frac{r}{a_0}\right) d\tau(\mathbf{y}), \tag{8}$$

with r being independent of the integration variable $\mathbf{y}$. The neglect of the retarded time variations which is incorporated in (8) has been discussed at length by Lighthill (1952) and Curle (1968). This neglect must not be performed in the analysis until (6) is derived or the answer of zero radiated sound results.

The time derivatives may be taken inside the integral sign of (8) and, to be sure, there are rather violent fluctuations in ρ_{tt} in a region of turbulence with combustion. In the form of (8), however, it is seen that the radiated sound due to F_3 depends upon the second time derivative of the mass in the control volume at a retarded time. One time derivative of this integral could be replaced, through mass conservation considerations, by a surface integral of the mass flux into the control volume. When this is done, one should recall the development leading to (6) which required the application of the divergence theorem and neglect of quantities depending upon surface integrals *outside* the violent region as compared with quantities dependent upon volume integrals through the active region. Therefore, since overall mass is conserved, (8) is expected to yield a rather inefficient sound source as compared with a pure monopole source as might occur if there were true mass sources in the interior of the region. Equation (8) will not yield the answer of zero radiated sound, however. It was, in fact, a term of this type investigated by Lighthill for either hot or cold jets. In the present case, however, the local density fluctuations become much more severe than in aerodynamic noise because of the presence of an energy source. For later order-of-magnitude considerations the magnitude of $\rho^{(3)}$ should be noted. For n correlation volumes, V_{cor} in V, consideration of the variance of the binomial distribution suggests

$$\rho^{(3)} \text{ is } O\left\{\frac{1}{4\pi a_0^2 r}\max[\rho_{tt}]\, n^{\frac{1}{2}} V_{\text{cor}}\right\}$$

with $n = V/V_{\text{cor}}$.

It remains to consider the contribution of F_2 to combustion noise. Following the same considerations leading to (8) a comparison of $\rho^{(2)}$ to $\rho^{(3)}$ yields

$$\rho^{(2)}/\rho^{(3)} \quad \text{is} \quad O\left[\frac{\max(p_{tt})}{a_0^2 \max(\rho_{tt})}\right].$$

If a locally laminar mechanism is responsible for propagation of a flame within a region of turbulence, there follows from momentum considerations

$$\frac{\max p_{tt}}{\max \rho_{tt}} \quad \text{is} \quad O\left[\frac{\rho_0}{\rho_1} S_L^2\right],$$

where ρ_0 and ρ_1 are the densities before and after the flame, respectively, and S_L is the laminar flame speed.

Roughly, then, $\rho^{(2)}/\rho^{(3)}$ is $O[S_L^2/a_0^2]$ which is approximately 10^{-6}. Consequently $\rho^{(2)} \ll \rho^{(3)}$ and may be neglected. Another alternative is that local explosions could take place over volumes in a time short compared to the time required to relieve the pressure (constant volume explosions). In such a case $\rho^{(2)}/\rho^{(3)}$ would be of the order of unity. But there appears to be no evidence that such events occur. Consequently, it appears that under current knowledge $\rho^{(3)}$ is the primary contributor to combustion generated noise.

Equation (8) will be investigated in more detail in the following developments. The important conclusion may already be drawn that in accord with experimental evidence (8) shows combustion noise to be due to a rather weak monopole acoustic source. The sound output is omni-directional. Further development requires knowledge of turbulent flame structure. Although the appropriate knowledge is meagre, an attempt is made below to estimate the sound output.

The acoustic power output is obtained by taking the time average of $(\rho^{(3)})^2$ and integrating $(\overline{\rho^2} a_0^3/\rho_0)$, the intensity, over a large sphere. Taking into consideration that ρ_{tt} values will only be correlated over distances of the order of a turbulent scale length and interchanging the order of integration yields the acoustic power

$$P = \frac{1}{4\pi\rho_0 a_0}\int_V d\tau(\mathbf{y})\int_{V_{\mathrm{cor}}} d\tau(\boldsymbol{\epsilon})\left[\overline{\rho_{tt}\left(\mathbf{y}, t-\frac{r}{a_0}\right)\rho_{tt}\left(\mathbf{y}+\boldsymbol{\epsilon}, t-\frac{r}{a_0}\right)}\right]. \quad (9)$$

The thermo-acoustic efficiency is

$$\eta_{ta} = P/\dot{m}fH, \quad (10)$$

where $\dot{m}$ is the total mass flow rate, f is the fuel mass fraction, and H is the heat of combustion.

It should be noted in (7) that one effect of a large combustion region is the possibility of a directional noise field. This effect, which will be investigated in a later paper, has important ramifications for noise radiated from large burners, such as turbojet afterburners.

3. Analysis of the combustion zone

Bragg's theory

Bragg's (1963) theory of combustion noise may be formally obtained from (9) and (10) by the following assumptions: (i) The propagation of the several flamelets constituting the turbulent flame is by a locally laminar mechanism. (ii) Estimates of the maximum value of ρ_{tt} are made by assignment of S_L/d_L to one time derivative, where d_L is a laminar flame thickness, and U/d_L to the other (where U is the flow velocity) while ordering the density change by $\Delta\rho$, the change across a laminar flame. (iii) The correlation volume is equal to d_L^3. (iv) In the time averaging procedure the mean square of ρ_{tt} is equal to its maximum value times the fraction of time which it may be attained, $(\dot{m}/V)/(\rho_0 S_L/d_L)$, which is the ratio of the turbulent flame volumetric consumption rate divided by the volumetric consumption rate in a laminar flame.

The result, presuming (9) and (10) yield

$$\eta_{ta} \text{ of } O\left[\left(\frac{1}{\dot{m}fH}\right)\left(\frac{1}{4\pi\rho_0 a_0}\right)(V)(V_{\mathrm{cor}})\,\overline{(\rho_{tt}\rho_{tt})}\right],$$

is
$$\eta_{ta} = \frac{K_B}{4\pi}\left(\frac{\Delta\rho}{\rho_0}\right)^2 \frac{U^2}{H}\frac{S_L}{a_0}\frac{1}{f} \tag{11}$$

which is Bragg's (1963) result up to a multiplicative factor of order unity, K_B. Of course, the original result was not arrived at through use of the present theoretical framework. Essentially, however, assumptions (i), (iii) and (iv) above were made in the analysis. Assumptions (ii) and (iii) appear difficult to justify, as will be seen below. In (10) there is no effect of turbulence intensity or scale. It is interesting, however, that the U and S_L scaling of η_{ta} lies within experimentally observed ranges.

Wrinkled laminar flame

There are two presently accepted mechanistic theories of the turbulent flame which are used as an aid in reasoning concerning turbulent flame structure. These theories are not particularly accurate, but they do explain certain features such as the turbulent flame speed S_t is greater than S_L and the turbulent flame is thicker than a laminar flame. As presented by Williams (1965) these two theories may be described as the wrinkled laminar flame theory and the distributed reaction theory. The former is generally used to describe propagation of a flame within large scale turbulence and the latter for fine scale turbulence. Considering first the wrinkled flame theory, the idea is that the only effect of turbulence is to wrinkle or distort the flame surface, which otherwise propagates in a locally laminar manner. By creating a larger surface area through distortion a greater consumption rate is possible than for a plane laminar flame front.

Focusing attention upon a position within a turbulent flame region, one would see a random passage of flames with time. The density change upon flame passage would correspond to that across a laminar flame, $\Delta\rho$. The speed of flame passage depends upon whether the flame is convected by the mean flow at speed U past the observer or perhaps propagates transverse to the mean flow past the observer at speed S_L. Note $U \gg S_L$ in usual turbulent flames and quite often S_t is not substantially greater than S_L so that the mean flame surface must be inclined to the mean flow at an angle $\theta = \arcsin S_t/U$ which may be reasonably small. In any event an element may be consumed only once during its passage through the turbulent flame zone and this places a limitation on the time between flame passages past the observer. A typical density trace might look as in figure 2. The values of ρ_{tt} are therefore ordered by $\Delta\rho(U^{1-q}S_L^q)^2/d_L^2$ where q is an exponent to be determined empirically. It is unlikely that it can be close to unity, however, because U is usually so large compared with S_L that the maximum time derivative of ρ is probably caused by convection of the flame past the observer. One expectation, however, is that q should decrease with an increase in U. The mean time between passage of flames, the most probable period in the random oscillations, should be equal to the time of passage of a fluid element through the turbulent flame zone, $T \approx d_t/S_t$. Calculating the ratio of observation time, during which important density changes are occurring, to the period

$$R = \frac{d_L/S_L^q U^{1-q}}{d_t/S_t} = \frac{S_t\rho_0 A_{\text{mean}} d_L}{d_t\rho_0 A_{\text{mean}} S_L^q U^{1-q}} = \frac{\dot{m} d_L}{V\rho_0 S_L^q U^{1-q}}.$$

It should be noted that the present estimate of R differs from that used by Bragg in assumption (iv) above; only if $q = 1$ are the statements identical. The present calculation appears to this writer to be preferred. In any event, the time average of ρ_{tt}^2 at a point, which is required in (9) appears to be ordered by

$$\overline{\rho_{tt}^2} \text{ is } O\{[\Delta\rho^2(U^{1-q}S_L^q)^4/d_L^4]R\} \text{ is } O\left\{(\Delta\rho)^2\left(\frac{U^{1-q}S_L^q}{d_L}\right)^3 \frac{\dot{m}}{V\rho_0}\right\}. \quad (12)$$

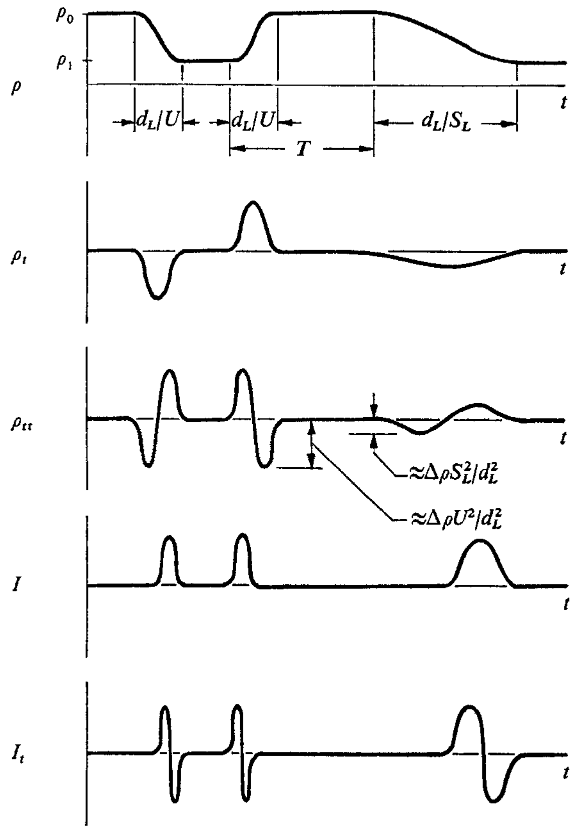

FIGURE 2. Expected local density and emission intensity histories and their time derivatives.

It is possible at this point to estimate the lower bound on frequency of the radiated noise as $\omega \approx 1/T$. Typically, d_t is observed to be of the order of l, the macroscopic burner dimension. A lower bound on S_t is S_L so that $\omega \geqslant S_L/l \approx 50$ Hz for $l = 1$ cm and $S_L = 50$ cm/sec. There would of course be many higher frequencies present, but this calculation verifies that combustion noise fills the lower portion of the frequency spectrum.

If an important variation in ρ is occurring, there is a volume element over which $\rho_{tt}(\mathbf{y})$ and $\rho_{tt}(\mathbf{y}+\boldsymbol{\epsilon})$ are correlated, V_{cor}. Here there are discrepancies in the literature. The mechanistic model adopted for the wrinkled flame would presume that each flamelet were correlated over an eddy. If l_t is a typical eddy dimension, $V_{\text{cor}} \approx d_L l_t^2$, the volume that a flame occupies if it is coherent over an eddy. On the other hand, as pointed out by Strehlow (1968), there are some experiments which indicate that the correlation volume is independent of the turbulence scale and dependent upon the cell size observed for so-called cellular flames (Markstein 1964). In this case $V_{\text{cor}} = K_c d_L^3$, since the cell size is proportional to the characteristic dimension of a laminar flame. It appears judicious at this point to choose

$$V_{\text{cor}} = K_{\text{cor}} d_L^{3-r} l_t^r, \tag{13}$$

with $2 \geqslant r \geqslant 0$ as an exponent to be empirically determined. Using (9), (10), (12) and (13) the thermo-acoustic efficiency is obtained.

$$\eta_{ta} = \frac{K_{WF}}{4\pi}\left(\frac{\Delta\rho}{\rho_0}\right)^2 \frac{1}{f}\left(\frac{l_t}{d_L}\right)^r \frac{(U^{1-q}S_L^q)^3}{a_0 H}, \tag{14}$$

where K_{WF} is a proportionality constant, incorporating K_{cor}. Since generally $l_t \propto l$, the characteristic burner dimension, and since $d_L \propto \alpha_0/S_L$, with α_0 the diffusivity of the gaseous reactants, (14) predicts

$$\eta_{ta} \propto U^{3-3q} S_L^{r+3q} l^r,$$

or

$$P \propto U^{4-3q} S_L^{r+3q} l^{2+r}. \tag{15}$$

This expression is derived presuming S_L may be varied independently of $\Delta\rho$. If this is not the case, as would be true if S_L were varied by varying mixture ratio, there is a slightly stronger dependence on S_L than given by (15). By comparison with (11) there are substantial differences in scaling rules for the present work as compared with Bragg's (1963) theory.

Comparing with experimental data, the work by Smith & Kilham (1963) appears the most exhaustive and reliable set of data for premixed flames. Their results indicate $P \propto U^2 S_L^m l^2$, where m is a function of Reynolds number varying from 3·4 to 0·6 and decreasing with a Reynolds number increase. Their length scaling indicates $r = 0$ and $q = \frac{2}{3}$, for a most reliable fit of the data. The value of q seems a bit high in view of the arguments above based upon physical reasoning. Nevertheless, a fit of the data is possible within the present framework. The results of Kotake & Hatta (1965), working with fuel rich premixed flames giving combined characteristics of premixed and diffusion flames, yielded $P \propto U^4 l^3$. In this case $q = 0$, $r = 1$ would be indicated. Smithson & Foster (1965) obtained $P \propto U^4$ which could also be explained by (15). Insufficient information was given in that work to investigate other scaling behaviour. The results of Giammar & Putnam (1970) are purposely being avoided here, because the flames were pure diffusion flames, and the present theory is clearly restricted to premixed flames. Further remarks concerning this point will follow.

Distributed reaction model

In a remarkable paper Hurle *et al.* (1968) showed that the fluctuations in C_2 and CH emission intensity, taken from ethylene-air premixed flames, could be related to the acoustic pressure by $p \propto dI/dt$, allowing for the time lag between acoustic generation and reception. The emission intensity, I, is measured from an image of the entire flame. This emission originates in the region of maximum reaction rate in a laminar flame and the experiments were performed to prove that combustion noise is monopole in nature, caused by the time rate of change of volume of small reacting pockets of combustibles. The physical picture of the origin of combustion noise has been presented differently in the present paper than by Hurle *et al.* and it is of interest to see if the present theory is compatible with those measurements. From (8) the acoustic density (or pressure) is proportional to the negative of a volume integral of ρ_{tt}. The expected derivative of local emission intensity of the radicals is shown in figure 2. This is drawn, of course, in only a qualitative fashion. It is clear from figure 2 that there is not a one-to-one correspondence between ρ_{tt} and the local $-dI/dt$ if the wrinkled flame model is adopted.

As put forth by John & Summerfield (1957), and others, there might be another model of the turbulent flame in which pockets of gas are not reacting according to a laminar flame mechanism but are reacting homogeneously. Transport of energy takes place not by molecular conduction but through turbulent diffusion (convection). Investigating this possibility consider the energy equation for low speed flows, neglecting molecular heat conduction and neglecting pressure variations.

$$\tilde{h}_{S_t} + \tilde{v}_i(\tilde{h}_S)_{x_i} = \tilde{\omega}H/\tilde{\rho}, \tag{16}$$

where $\tilde{h}_S$ is the sensible enthalpy, $\tilde{\omega}$ is a global reaction rate function expressing the fuel conversion rate per unit volume and the $\sim$ denotes complete quantities, not their fluctuating counterparts. Assuming no substantial molecular weight differences between products and reactants and assuming a thermally and calorically perfect gas, it is found that (16) may be written

$$\tilde{\rho}_t + v_i\tilde{\rho}_{x_i} = -(\gamma-1)(H/a^2)\tilde{\omega}, \tag{17}$$

where γ is the ratio of specific heats and a is the local speed of sound. Using the continuity equation and (17), the velocity divergence is directly related to the reaction rate by

$$(\tilde{v}_i)_{x_i} = (\gamma-1)(H/\gamma\tilde{p})\tilde{\omega}. \tag{18}$$

Since γ, H and $\tilde{p}$ are constants, (18) is valid for the fluctuations in $\tilde{v}_{x_i}$ and $\tilde{\omega}$. Thus, from the physical interpretation of a divergence, the picture of 'expanding balloons' so often used as the molopole analogue and used by Bragg (1963), Gaydon & Wolfhard (1953), Hurle *et al.* (1968) and others seems plausible as the physical explanation of combustion noise. Recall, however, that to reach this interpretation the flame propagation must not be by a laminar mechanism. In fact, as shown by Fendell (1967) in a region of laminar propagation $\tilde{\omega}$ would be essentially balanced by the molecular conduction term, not by the velocity divergence. This remark rests, however, upon a one-dimensional treatment. It is not obvious whether valid conclusions regarding the balance of $\tilde{\omega}$ and molecular conduction may be drawn from a one-dimensional analysis. Since the properties of unsteady, non-planar laminar flame fronts have not been adequately analysed as yet, the statement that the 'expanding balloon' picture rests upon rejection of a laminar mechanism must be taken as a conjecture, with incomplete supporting evidence. It, however, still remains to show that (18) leads to an explanation of the results of Hurle *et al.* (1968).

Integrating (18) over the combustion volume, there is obtained

$$\int_V v_{x_i}\left(\mathbf{y}, t-\frac{r}{a_0}\right) d\tau(\mathbf{y}) = \int_S v_i n_i d\sigma = (\gamma-1)\frac{H}{\gamma\tilde{p}}\int_V \omega\left(\mathbf{y}, t-\frac{r}{a_0}\right) d\tau(\mathbf{y})$$

or

$$\rho_0 \int_S v_i n_i d\sigma = \frac{\gamma-1}{a_0^2} H \int_V \omega\left(\mathbf{y}, t-\frac{r}{a_0}\right) d\tau(\mathbf{y}), \tag{19}$$

where $d\sigma$ is an element of the surface of the control volume and n_i is the outward unit normal vector. Now return to (8) and use the continuity equation to obtain

$$\rho = +\frac{1}{4\pi a_0^2 r}\frac{\partial}{\partial t}\int_S \tilde{\rho}\tilde{v}_i n_i d\sigma. \tag{20}$$

Since there are no violent fluctuations in $\tilde{\rho}$ on S, $\tilde{\rho}$ may be considered constant on the inner and outer flame surfaces. Considering $\rho_0 > \rho_1$, and assuming that there is no preferred direction for the velocity fluctuations, it follows from (20) and (19) that

$$\rho = \frac{(\gamma-1)H}{4\pi a_0^4 r}\int_V \omega_t\left(\mathbf{y}, t-\frac{r}{a_0}\right) d\tau(\mathbf{y}). \tag{21}$$

The error made in (21) is $O(\rho_1/\rho_0)$ compared to unity. Thus, the quite interesting result is obtained that the work of Hurle *et al.* (1968) may be explained, but not through the wrinkled flame model.

Proceeding to an estimate of η_{ta} and P the analogue of (9) is

$$P = \frac{(\gamma-1)^2 H^2}{4\pi a_0^5 \rho_0}\int_V d\tau(\mathbf{y})\int_{V_{\mathrm{cor}}} d\tau(\boldsymbol{\epsilon})\left[\overline{\omega_t\left(\mathbf{y}, t-\frac{r}{a_0}\right)\omega_t\left(\mathbf{y}+\boldsymbol{\epsilon}, t-\frac{r}{a_0}\right)}\right]. \tag{22}$$

ω and changes in ω are ordered by $\rho_0 S_L/d_L$. Since d_L/S_L is usually much smaller than l_t/U, the local time derivative $\partial/\partial t$ depends upon the convection of reacting eddies past the observer, U/l_t. The intermittency of passing eddies determines the base period which is the same as with the wrinkled flame model, $T = d_t/S_t$. Consequently

$$\overline{\omega_t^2} \text{ is } O\left[\left(\frac{\rho_0 S_L U}{d_L l_t}\right)^2 \frac{\dot{m} d_L}{V\rho_0 U}\right]. \tag{23}$$

Note in this model that q does not enter since its origin is based upon flame movement by the observer through a laminar propagation mechanism. In the present case V_{cor} is $O(l_t^3)$ and the result for η_{ta} follows from (10), (22) and (23).

$$\eta_{ta} = \frac{K_{DR}}{4\pi}\frac{(\gamma-1)^2}{f}\frac{HUS_L^2}{a_0^5}\frac{l_t}{d_L}. \tag{24}$$

Approximately, $c_p(T_1 - T_0) = H$ so that (24) becomes

$$\eta_{ta} = \frac{K_{DR}}{4\pi}\frac{\Delta\rho}{\rho_1}\frac{\gamma-1}{f}\frac{US_L^2}{a_0^3}\frac{l_t}{d_L}. \tag{25}$$

Now with no adjustable constants (25) yields

$$\eta_{ta} \propto US_L^3 l,$$

$$P \propto U^2 S_L^3 l^3.$$

It is remarkable that this result has the proper velocity scaling and reasonable S_L and l behaviour. Relation (25) also shows a different dependence upon $\Delta\rho$ from (14), a fact which could be experimentally checked. Note also that H is absent from (25).

Because of the agreement with the work of Hurle *et al.*, (25) is preferred by this writer to (14), even though the scaling rules are not in precise agreement with those obtained by Smith & Kilham (1963). There is another reason for this preference which will become clear upon numerical comparison. It is interesting, however, that the work of Hurle *et al.* was carried out in a régime where the wrinkled laminar flame model would be expected to be more valid than a distributed reaction model. Furthermore those authors considered their work as indication that the wrinkled flame model was valid. In reality both distributed and locally laminar reactions probably take place and the proper description of combustion noise would include contributions from both (14) and (25). It should be cautioned, however, that (25) cannot explain the U^4 scaling obtained by Smithson & Foster (1965) and Kotake & Hatta (1965).

There is another model of the turbulent premixed flame that should be given consideration in future work. The model of Shelkin (1947), which considers a wrinkled laminar flame that continuously sheds pockets of unburned gas which burn in an extended zone behind the main front, would seem to combine the features of both analyses presented here. Furthermore, examination of Shelkin's (1947) theory shows that an effect of the turbulence intensity would enter an expression for the noise output. Such a dependence is absent from (14) and (25).

Numerical verification

If the theories have been carried out properly there is the usual, almost mystical, expectation that K_{WF} and K_{DR} should turn out to be constants not too far removed from unity. Comparing with the Smith & Kilham (1963) results, the following numerical values are chosen

$$q = \tfrac{2}{3} \text{ and } r = 0, \qquad S_L = 50\,\text{cm/sec},$$
$$d_L = 0\cdot 1\,\text{cm}, \qquad U = 4500\,\text{cm/sec},$$
$$f = 0\cdot 06, \qquad a_0 = 3\times 10^4\,\text{cm/sec},$$
$$H = 4\cdot 2\times 10^{11}\,\text{cm}^2/\text{sec}^2 = 10^4\,\text{cal/g} = 18000\,\text{B.Th.U./lb},$$
$$\Delta\rho/\rho_1 = 3, \qquad \eta_{ta} = 8\cdot 2\times 10^{-8},$$
$$l_t = 0\cdot 2l, \qquad l = 0\cdot 5\,\text{cm},$$
$$\gamma = 1\cdot 3.$$

This case corresponds roughly to 6·0 % ethylene-air at a Reynolds number of 25,000 in a 0·25 in. diameter burner tube. The results for K_{WF} and K_{DR} are

$$K_{WF} = 124, \quad K_{DR} = 0\cdot 17.$$

The reason mentioned above that the distributed reaction model seems preferred is that K_{DR} is closer to the expected value of unity. This is clearly not a proof, only a preference.

Diffusion flames

There does not appear sufficient diffusion flame noise data in a simple jet configuration to warrant an attempt at a theoretical construction at this time. The data of Kotake & Hatta (1965) were taken with a combination premixed-diffusion flame. The data of Giammar & Putnam (1970) were from an impinging jet situation or a complex 'octopus' burner, consisting of eight impinging jets. Furthermore, the scaling rules obtained by the two sets of workers are completely different. In addition it should be pointed out that the theory for diffusion flames would entail an additional complication over that of premixed flames. Obviously, the turbulent mixing, necessary before reaction can take place, would play a part in determination of the noise output because it must in some manner determine the size of the reacting gas pockets. In any event, further exploratory experimentation in a simple configuration is required before a theoretical formulaton can be attempted.

4. Concluding remarks

Two theories have been constructed for combustion noise radiated from premixed flames. These theories follow rigorously from the principles of fluid mechanics coupled with physical reasoning concerning turbulent flame structure. The results of the theories are able to quite well correlate one body of extensive data on premixed flames, and reasonable estimates of combustion noise output can be made. The body of data correlated covers circular burners with diameters from 0·25 in. to 0·5 in., flow velocities from 20 to 200 ft/sec and the fuels ethylene, propylene and propane. The spectral content of the noise output has been estimated and found to correspond to the experimental fact of low-frequency noise or 'roar'. In a more general sense the present theory is able to explain the observed facts that combustion noise power output does not scale with velocity to an exponent higher than four nor with the characteristic flow dimension to an exponent higher than three. Furthermore, the origin of the apparent monopole source that represents combustion noise has been isolated.

There is a clear need for more data on premixed flame noise. A larger range of burner sizes, flow velocities and laminar flame speeds should be covered than has been in the past. Grid induced turbulence should be investigated to better define the effect of intensity and scale upon the combustion noise output. It will be noted that in neither (14) or (25) did any intensity effect enter the theory. Critical tests should be performed to determine which, if either, expression best represents combustion noise or whether a judicious combination of the two can be accomplished. A series of experiments should be run using the emission technique of Hurle *et al.* while detailed noise scaling data are also obtained.

Of practical interest is the fact that if r is non-zero or the distributed reaction model is valid there is an adverse scaling of combustion noise with burner size. This may be of interest in the case of larger afterburning turbojets. Furthermore, if the wrinkled flame model is valid and q decreases as U increases there is an adverse scaling of noise output with U. Again the afterburner problem arises because of the high speed flows in the burner section. Finally, there is the interesting question of increased directionality of output as burner size increases. This was mentioned in connection with (7). An obvious practical case of concern is again an afterburner application. Flames held by flameholders are typically elongated and may have significant dimensions in such an application.

REFERENCES

BRAGG, S. L. 1963 *J. Inst. Fuel*, **36**, 12.

BRIFFA, F. E. J. & FURSEY, R. A. E. 1967 *Nature*, **214**, 75.

CROCCO, L. & CHENG, S. I. 1956 *Theory of Combustion Instability in Liquid Propellant Rocket Motors*. London: Butterworths.

CURLE, S. N. 1968 *Noise and Acoustic Fatigue in Aeronautics* (ed. E. J. Richards and D. J. Mead). London: Wiley.

DOAK, P. E. 1970 Submitted to *J. Sound and Vibration*.

FENDELL, F. E. 1967 *AIAA J* **5**, 808.

GAYDON, A. G. & WOLFHARD, H. G. 1953 *Flames*, 1st edn. London: Chapman and Hall.

GIAMMAR, R. D. & PUTNAM, A. A. 1970 *J. Engineering for Power*, 157.

HURLE, I. R., PRICE, R. B., SUGDEN, T. M. & THOMAS, A. 1968 *Proc. Roy. Soc.* A **303**, 409.

JOHN, R. R. & SUMMERFIELD, M. 1957 *Jet Propulsion*, **27**, 169.

KARLOVITZ, B. 1951 *J. Chem. Phys.* **19**, 541.

KOTAKE, S. & HATTA, K. 1965 *Bulletin JSME*, **8**, 24.

LIGHTHLIL, M. J. 1952 *Proc Roy. Soc.* A **211**, 564.

LIGHTHILL, M. J. 1954 *Proc. Roy. Soc.* A **222**, 1.

MARKSTEIN, G. H. 1964 *Nonsteady Flame Propagation*. New York: Macmillan.

RIBNER, H. S. 1964 *Adv. Appl. Mech.* **8**, 103.

SHELKIN, K. I. 1947 *NACA Tech. Memo.* 1110.

SMITH, T. J. B. & KILHAM, J. S. 1963 *J. Acous. Soc. Am.* **35**, 715.

SMITHSON, R. N. & FOSTER, P. J. 1965 *Combustion and Flame*, **9**, 426.

STREHLOW, R. A. 1968 *Fundamentals of Combustion*. Scranton, Pa.: International Textbook.

WILLIAMS, F. A. 1965 *Combustion Theory*. Reading, Mass.: Addison-Wesley.